INVESTIGATION OF HEAT TRANSFER IN A REGION OF GRADIENT FLOW WHEN A PLANE TURBULENT JET IS INCIDENT ON A FLAT PLATE NORMAL TO THE FLOW

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The boundary conditions are determined experimentally for a system of differential equations describing heat transfer in a region of gradient flow when a plane turbulent jet is incident on a flat plate normal to the flow. The analytical solution is compared with direct measurements.

When a plane jet is incident on a flat plate normal to the direction of its velocity, as for an axisymmetric jet [1], we can distinguish three flow regions:

1. A region of gradient flow where the pressure falls along the plate from a maximum on the line where the flow spreads out to nearly atmospheric at a distance x from that line. The velocity at the upper boundary of the boundary layer, U, increases from zero on the line to a maximum value U_* at a distance x_* from it. The y-axis is in the plane of symmetry of the jet normal to the plate and the x-axis is along the plate normal to the line where the flow spreads out.

In this region we can assume that the flow beyond the limit of the boundary layer is potential and that it is described by the following equation:

$$U\frac{\partial U}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}. \tag{1}$$

2. A region of transitional flow where the velocity U remains practically constant.

3. A region of fundamental flow where the velocity U begins to decrease due to the braking effect of the jet at the wall, while the pressure remains practically constant.

In this paper we give the results of investigating only the region of gradient flow. From the practical point of view this region is of greatest interest. At the same time the physical processes are most complex here and their investigation is very time-consuming.

Noting that the Reynolds numbers for this region of the flow are small, we can assume that the boundary layer is laminar here, particularly because $dP/dx < 0$.

Under these conditions the system of differential equations for the motion, and of continuity and energy, and the boundary conditions can be written as:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{3}$$

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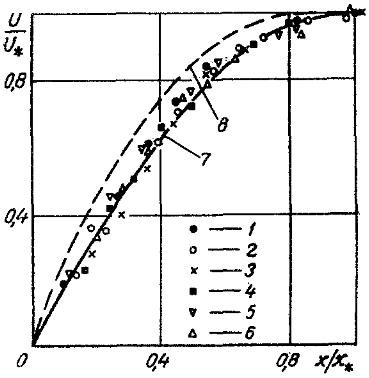

Fig. 1. The behavior of the velocity at the outer edge of the boundary layer: b_0 = 31 mm; u_0 = 15.5 m/sec; 1) $\bar{h}$ = 1.67; 2) 4; b_0 = 22 mm; u_0 = 17 m/sec; 3) $\bar{h}$ = 6.25; 4) 9.1; b_0 = 11 mm; u_0 = 20.8 m/sec; 5) h = 17.8; 6) 22.3; 7) from Eq. (15); 8) from the data of [6].

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = a \frac{\partial^2 T}{\partial y^2}. \tag{4}$$

The boundary conditions are:

for $y = 0$, $u = v = 0$, $\bar{T} = 0$;

for $y = \delta$, $u = U$; for $y = \delta_t$, $\bar{T} = 1$.

The velocity at the outer edge of the boundary layer can be written in the form of two terms of a series which adequately reflects the actual conditions (Fig. 1)

$$U = \beta_1 x + \beta_2 x^3 + \ldots, \tag{5}$$

where β_1 and β_2 are constants, depending on $\bar{h}$. The equation of continuity is satisfied if

$$u = \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x}. \tag{6}$$

We introduce the nondimensional coordinate $\eta = y\sqrt{\frac{\beta_1}{\nu}}$ and define the stream function Ψ as follows:

$$\Psi = \sqrt{\frac{\nu}{\beta_1}}\,[\beta_1 x f_1(\eta) + 4\beta_3 x^3 f_3(\eta) + \ldots], \tag{7}$$

where $f_i(\eta)$ is a function of the nondimensional coordinate η.

From (6) and (7) we find the velocity distributions for u and v through the thickness of the boundary layer in the form

$$u = \beta_1 f_1'(\eta)\, x + 4\beta_3 f_3'(\eta)\, x^3 + \ldots, \tag{8}$$

$$v = -\sqrt{\frac{\nu}{\beta_1}}\,[\beta_1 f_1(\eta) + 12\beta_3 f_3(\eta)\, x^2 + \ldots], \tag{9}$$

where the primes indicate differentiation with respect to η. The nondimensional temperature distribution through the thickness of the boundary layer can be written as [4]

$$\bar{T}(x, y) = \frac{T - T_w}{T_\infty - T_w} = F_0(\eta) + \frac{\beta_3}{\beta_1} x^2 F_2(\eta) + \ldots, \tag{10}$$

where $F_0(\eta)$, $F_2(\eta)$, . . . are functions of the nondimensional coordinate η and of Pr.

Substituting (8) and (9) in (2) and (10) in (4), we obtain, after some transformations, the following differential equations [4, 7]:

$$f_1'^2 - f_1 f_1'' = 1 + f_1''',$$
$$4f_1' f_3' - 3f_1'' f_3 - f_1 f_3'' = 1 + f_3''',$$
$$\ldots\ldots\ldots\ldots\ldots$$

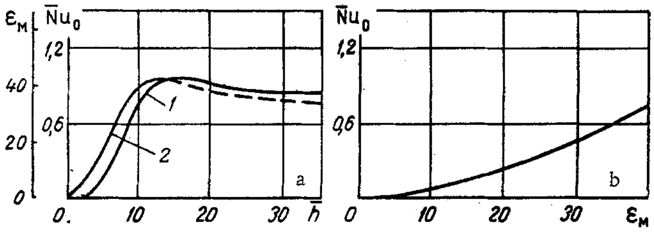

Fig. 2. The effect of turbulence on the intensification of heat transfer on the line where the flow spreads out. a: 1) Nu_0 as a function of $\bar{h}$; 2) ε_M as a percentage of h [6]; b) Nu_0 as a function of ε_M.

$$\frac{1}{\Pr} F_0'' + f_1 F_0' = 0, \tag{11}$$

$$\frac{1}{\Pr} F_2'' + f_1 F_2' - 2 f_1' F_2 = -12 f_3 F_0',$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

with boundary conditions of the form

$$\text{for } \eta \to 0 \quad f_1 = f_1' = f_3 = f_3' = 0, \quad F_0 = F_2 = 0, \quad F_4 = 0, \quad f_5 = f_5' = 0;$$

$$\text{for } \eta \to \infty \quad f_1' = 1, \quad f_3' = \frac{1}{4}, \quad F_0 = 1, \quad F_2 = 0, \quad F_4 = 0, \quad f_5' = \frac{1}{6}.$$

We solve Eqs. (11) with the above boundary conditions numerically [2, 4].

The values of the unknowns f_i, f_i', f_i'', f_i''' are given in [3], while F_i' is given in [4]. The heat flow at the wall can be determined from the Fourier–Newton law; equating the absolute values of the right sides, we obtain

$$\lambda \left(\frac{\partial T}{\partial y} \right)_{y=0} = \alpha (T_\infty - T_w), \tag{12}$$

from which, noting (10), we have†

$$\alpha = \lambda \sqrt{\frac{\beta_1}{\nu}} \left[F_0'(0) + \frac{\beta_3}{\beta_1} F_2'(0) x^2 + \ldots \right]. \tag{13}$$

For Pr = 0.7, the values of the functions are $F_0'(0) = 0.4959$, $F_2'(0) = 0.4476$ [4]. Noting these values, we can write (13) as:

$$\mathrm{Nu}_x = b_0 \sqrt{\frac{\beta_1}{\nu}} \left(0.496 + 0.448 x^2 \frac{\beta_3}{\beta_1} + \ldots \right). \tag{14}$$

Equation (14) can be used in computations if we know the constants β_1 and β_3. They were determined experimentally. The experiments were made with pipes of gap length 150 mm and width 5, 11, 20, 31 mm. The polished flat plate had dimensions 200 × 700 mm; the holes to determine the static pressure were of diameter 0.3 mm, and the distance between them was 5 mm. The total pressure at the wall boundary layer was measured by a specially made dynamic head meter. The experiments were made at flow velocities at the nozzle edge from 5-25 m/sec.

The experimental results can be put in the form of the following universal equation (Fig. 1):

$$\frac{U}{U_*} = 1.6 \frac{x}{x_*} - 0.6 \left(\frac{x}{x_*} \right)^3. \tag{15}$$

†We retain only the first two terms of the series.

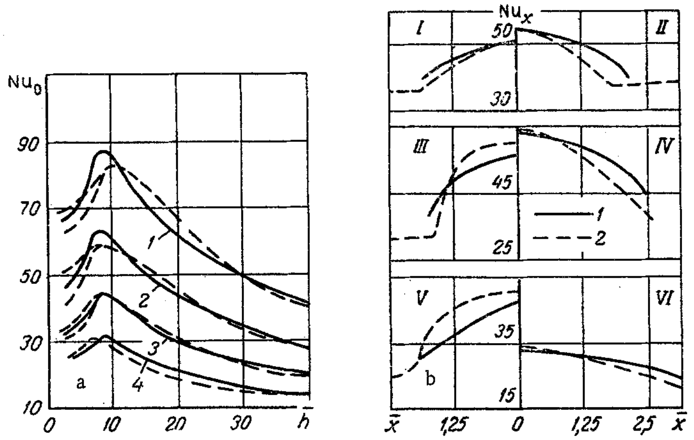

Fig. 3. Heat-transfer coefficients in the region of gradient flow: a) Nu_0 as a function of $\bar{h}$ [1-4 (continuous curves) from Eqs. (24)-(26); 1) Re_0 = 22,000; 2) 11,000; 3) 5500; 4) 2750]; dotted curves) from the experimental data of [5]; b) Nu as a function of $\bar{x}$ (Re_0 = 11,000); I) $\bar{h}$ = 2; II) 5; III) 6; IV) 8; V) 16; VI) 32); 1) from Eqs. (24)-(26); 2) from [5].

Here U_* and x_* depend on the distance between the nozzle edge and the flat plate. To determine them from the measured results we constructed the following equations:

for $1 \le \bar{h} \le 6.5$

$$\frac{U_*}{U_0} = \frac{1}{\bar{h}^{0,1}}; \quad \bar{x}_* = \frac{x_*}{b_0} = 1.7\bar{h}^{0,1}, \tag{16}$$

for $\bar{h} \ge 6.5$

$$\frac{U_*}{U_0} = \frac{2.3}{\bar{h}^{0.5}}; \quad \bar{x}_* = \frac{x_*}{b_0} = 0.58\bar{h}^{0.7}. \tag{17}$$

Comparing (5) and (14), we find that in the range of conditions investigated

$$\beta_1 = 1.6\frac{U_*}{x_*}, \quad \beta_3 = 0.6\frac{U_*}{x_*^3}. \tag{18}$$

The computational equations for determining the local values of the coefficients of heat transfer α in the region of gradient flow are found by substituting (18) in (13) and noting (16) and (17), i.e.,

for $1 \le \bar{h} \le 6.5$

$$Nu_x = 0.48\, Re_0^{0.5}\left(1 - 0.116\frac{\bar{x}^2}{\bar{h}^{0.2}}\right)\frac{1}{\bar{h}^{0.1}}, \tag{19}$$

for $\bar{h} \ge 6.5$

$$Nu_x = 1.25\, Re_0^{0.5}\left(1 - 1.05\frac{\bar{x}^2}{\bar{h}^{1.4}}\right)\frac{1}{\bar{h}^{0.6}}. \tag{20}$$

On the line where the flow spreads out, when $\bar{x} = 0$, Eqs. (19) and (20) take the form:

for $1 \le \bar{h} \le 6.5$

$$Nu_0 = 0.48\, Re_0^{0.5}\bar{h}^{-0.1}, \tag{21}$$

for $\bar{h} \ge 6.5$

$$Nu_0 = 1.25\, Re_0^{0.5}\bar{h}^{-0.6}. \tag{22}$$

Comparison of the coefficients of heat transfer α computed from (21) and (22) with direct measurements [6] showed that:

1. For values of $\bar{h} < 3$ good agreement is observed. The correctness of the comparison is confirmed by the good agreement between the two equations $\bar{U} = f(\bar{x})$ (Fig. 1); the first, obtained experimentally in this paper, and the second, obtained by calculation from (1) from data on the pressure distribution with respect to x taken from Gardon and Akfirat [5].

We note that when $\bar{h} < 14$ a certain scatter in the measured values is observed as a function of the width b_0 of the gap. The dimension b_0 of the gap affects the initial degree of turbulence ε_0 of the jet. Smaller values of α correspond to smaller degrees of initial jet turbulence.

To ensure that the comparison was made for the same initial degree of turbulence, the computed values of α from (21) were compared with the smaller values obtained experimentally.

2. When $\bar{h} \geq 3$, the measured values of α become greater than the computed ones; as $\bar{h}$ increases, the difference between the measured and the computed values of the heat-transfer coefficients increases and reaches a maximum value when $\bar{h} \approx 14$.

3. As $\bar{h} > 14$ increases further, the difference becomes constant.

In the neighborhood of the line where the flow spreads out, the measured values of the heat-transfer coefficients α_0' are approximately twice the computed values of α_0 when $\bar{h} > 14$. To explain this large discrepancy, we consider the curves in Fig. 2a and b.

There are two curves in Fig. 2a. The first (1) shows the equation $\overline{\mathrm{Nu}}_0 = f(\bar{h})$ on the line where the flow spreads out for the value of α_0 computed from (20)-(21) at increasing distances of the plate from the nozzle. The second (2) shows the equation $\varepsilon_M = f(h)$ [5], the local axial degree of jet turbulence at increasing distances from the nozzle, the dotted part of curve 2 being constructed on the basis that when $\bar{h} > 14$, ε_M, as measured by the authors, remains virtually constant. The two curves were obtained with the same nozzle at Re = 5500. The value of Nu_0 was determined from the equation

$$\overline{\mathrm{Nu}}_0 = \frac{\mathrm{Nu}_0'}{\mathrm{Nu}_0} - 1. \tag{23}$$

The good correlation between curves 1 and 2 in Fig. 2a permits the following conclusions:

1. The increase in the heat-transfer coefficient α_0' at the plate for increasing distance from the nozzle is associated with the degree of turbulence ε_M of the incident jet.
2. In the first approximation this relation can be taken to be linear.

Figure 2b shows the equation $\overline{\mathrm{Nu}}_0 = f(\varepsilon_M)$, which was obtained by cross plotting the curves of Fig. 2a.

From the relations shown in Fig. 2a and b, and assuming that a correction to the equations for the effect of the degree of turbulence in the neighborhood of the line where the flow spreads out is valid throughout the whole region of the gradient flow, we can obtain improved equations in the form:

for $1 \leq \bar{h} \leq 6.5$

$$\mathrm{Nu}_x = 0.48\frac{\mathrm{Re}_0^{0.5}}{\bar{h}^{0.1}}\left(1 - 0.116\frac{\bar{x}^2}{\bar{h}^{0.2}}\right)(1 + 0.015\,\varepsilon_M), \tag{24}$$

for $6.5 \leq \bar{h} \leq 12$

$$\mathrm{Nu}_x = 1.25\frac{\mathrm{Re}_0^{0.5}}{\bar{h}^{0.6}}\left(1 - 1.05\frac{\bar{x}^2}{\bar{h}^{1.4}}\right)(1 + 0.019\varepsilon_M), \tag{25}$$

for $h \geq 12$

$$\mathrm{Nu}_x = 1.25\frac{\mathrm{Re}_0^{0.5}}{\bar{h}^{0.6}}\left(1 - 1.05\frac{\bar{x}^2}{\bar{h}^{1.4}}\right)(1 + 0.025\varepsilon_M). \tag{26}$$

The computational results from Eqs. (24)-(26) are shown in Fig. 3a and b by continuous lines; the experimental results are shown by dotted lines. Comparison of the computed heat-transfer coefficients with the experimental ones permits the following conclusion: in spite of the fact that the correction equations (24)-(26) were applied for only one Reynolds number $Re_0 = 5500$, and one nozzle, with initial degree of turbulence $\varepsilon_0 \sim 1\%$, the computed values agree with the experimental ones (the discrepancy does not exceed 15%) over a wide range of Reynolds numbers up to $Re_0 = 22{,}000$ and nozzles with initial degree of turbulence up to $\varepsilon_0 = 7\%$.

NOTATION

b_0	is the nozzle gap width;
h	is the distance from nozzle edge to plate;
$\bar{h} = h/b_0$	is the nondimensional form of h;
x	is the current abscissa;
$\bar{x} = x/b_0$	is the nondimensional abscissa;
$\bar{x}_* = x_*/b_0$	is the nondimensional abscissa at which the velocity at the outer edge of the boundary layer reaches its maximum value;
U	is the velocity at the outer edge of the boundary layer;
U_*	is the maximum velocity at the outer edge;
$\bar{U}_*$	is the maximum velocity in nondimensional form;
T_W	is the surface temperature of the plate;
T_∞	is the temperature of the incident flow;
$T = (T - T_W)/(T_\infty - T_W)$	is the nondimensional temperature;
p	is the static pressure at a given section of the boundary layer;
δ	is the dynamic boundary layer thickness;
δ_T	is the thermal boundary layer thickness;
α	is the heat-transfer coefficient;
$Re_0 = u_0 b_0/\nu$	is the Reynolds number referred to parameters at nozzle edge;
$Nu = \alpha b_0/\lambda$	is the Nusselt number;
$\varepsilon_0 = u'/u_0$	is the degree of initial turbulence;
ε_M	is the degree of turbulence on the jet axis referred to the local axial velocity.

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